Space Complexity and Intractability

Space Complexity

- Space Complexity
  - Measure of the amount of working storage an algorithm needs
  - How much memory (worst case) is needed at any point in the algorithm.
  - Uses big-O notation
    - Upper bounds on necessary space as input grows

DEFINITION 8.1

Let M be a deterministic Turing machine that halts on all inputs. The ***space complexity*** of M is the function $f\colon \mathcal{N} \longrightarrow \mathcal{N}$, where $f(n)$ is the maximum number of tape cells that M scans on any input of length n. If the space complexity of M is $f(n)$, we also say that M runs in space $f(n)$.

If M is a nondeterministic Turing machine wherein all branches halt on all inputs, we define its space complexity $f(n)$ to be the maximum number of tape cells that M scans on any branch of its computation for any input of length n.

Space Complexity Notes

- Time and space complexity are separate
- Unlike time, space is reusable during runtime.

DEFINITION 8.2

Let $f: \mathcal{N} \longrightarrow \mathcal{R}^+$ be a function. The ***space complexity classes***, **SPACE($f(n)$)** and **NSPACE($f(n)$)**, are defined as follows.

$$\text{SPACE}(f(n)) = \{L \mid L \text{ is a language decided by an } O(f(n)) \text{ space deterministic Turing machine}\}.$$

$$\text{NSPACE}(f(n)) = \{L \mid L \text{ is a language decided by an } O(f(n)) \text{ space nondeterministic Turing machine}\}.$$

- When using a deterministic TM to simulate a nondeterministic TM
  - Can require an exponential increase in time
  - However, only a small increase for space is required.
- Savitch's Theorem: nondeterministic TM uses f(n) then the equivalent deterministic TM will use $f^2(n)$

THEOREM 8.5

Savitch's theorem For any[1] function $f: \mathcal{N} \longrightarrow \mathcal{R}^+$, where $f(n) \geq n$,

$$\text{NSPACE}(f(n)) \subseteq \text{SPACE}(f^2(n)).$$

PSPACE

DEFINITION 8.6

PSPACE is the class of languages that are decidable in polynomial space on a deterministic Turing machine. In other words,

$$\text{PSPACE} = \bigcup_k \text{SPACE}(n^k).$$

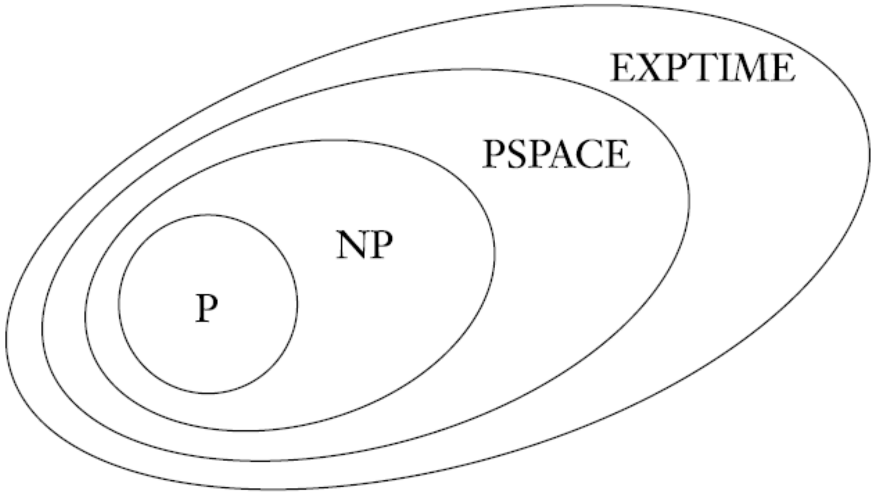

FIGURE 8.7
Conjectured relationships among P, NP, PSPACE, and EXPTIME