Turing Machines Examples

Example 1

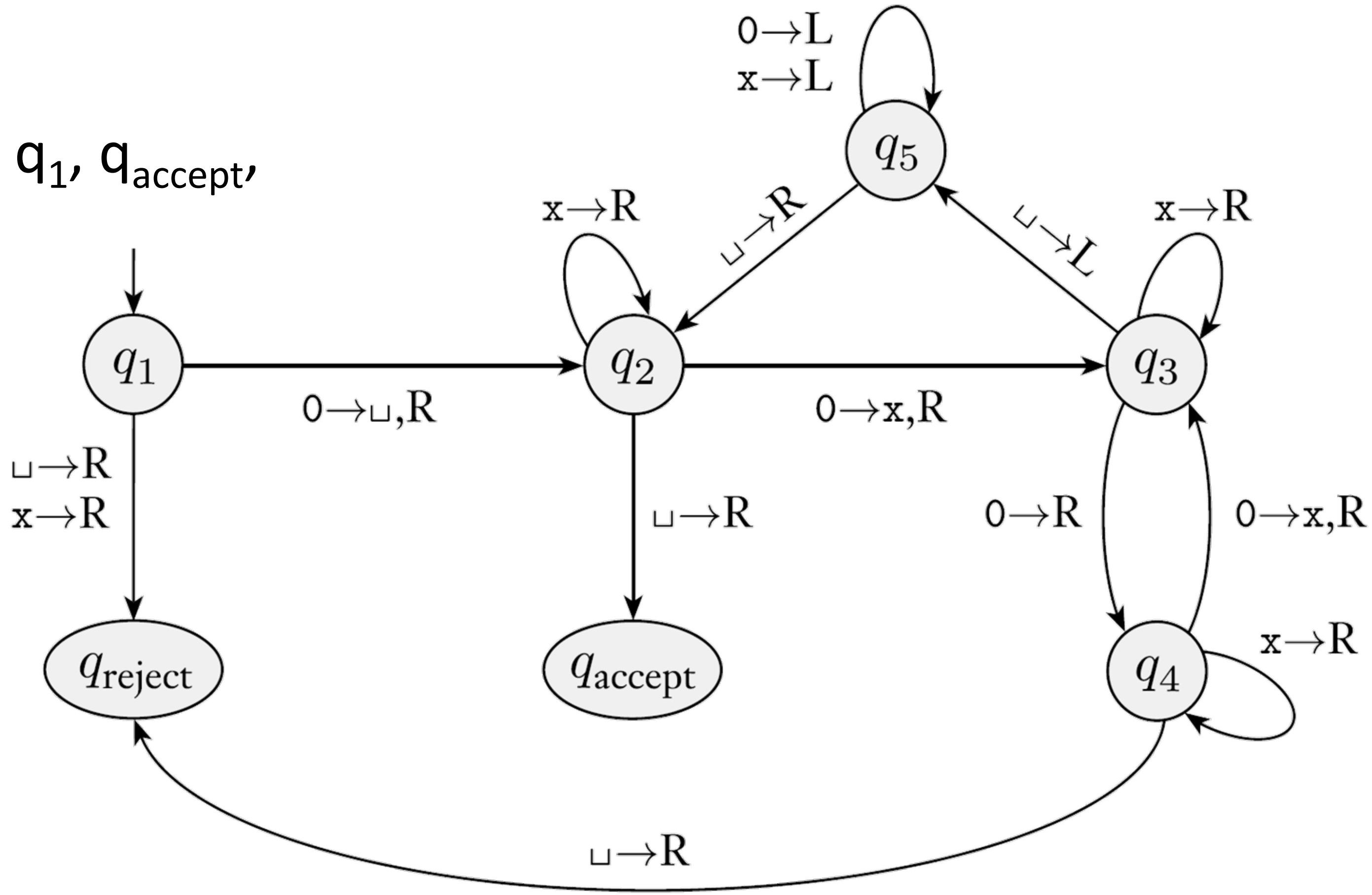
- Given TM M_2 that decides A = { 0^{2^n} | n ≥ 0 }
  - Language consists of strings of 0's whose length is a power of 2

- M_2 = "On input string w:
  1. Sweep left to right across the tape, crossing off every other 0
  2. If in stage 1 the tape contained a single 0, accept
  3. If in stage 1 the tape contained more than a single 0 and the number of 0s was odd, reject
  4. Return the head to the left-hand end of the tape
  5. Go to stage 1

- At each iteration of stage 1, the number of 0s is cut in half
  - Done by marking every other 0
  - Keeps track of number of 0s on each pass

Example 1

- Formal Definition, $M_2 = (Q, \Sigma, \Gamma, \delta, q_0, qacc_{ept}, qrej_{ect})$
  - $Q = \{q_1, \dots, q_5, q_{accept}, q_{reject}\}$
  - $\Sigma = \{0\}$
  - $\Gamma = \{0, \mathrm{x}, \sqcup\}$
  - We describe δ with a state diagram
  - The start, accept, and reject states are q_1, q_{accept}, and q_{reject}, respectively

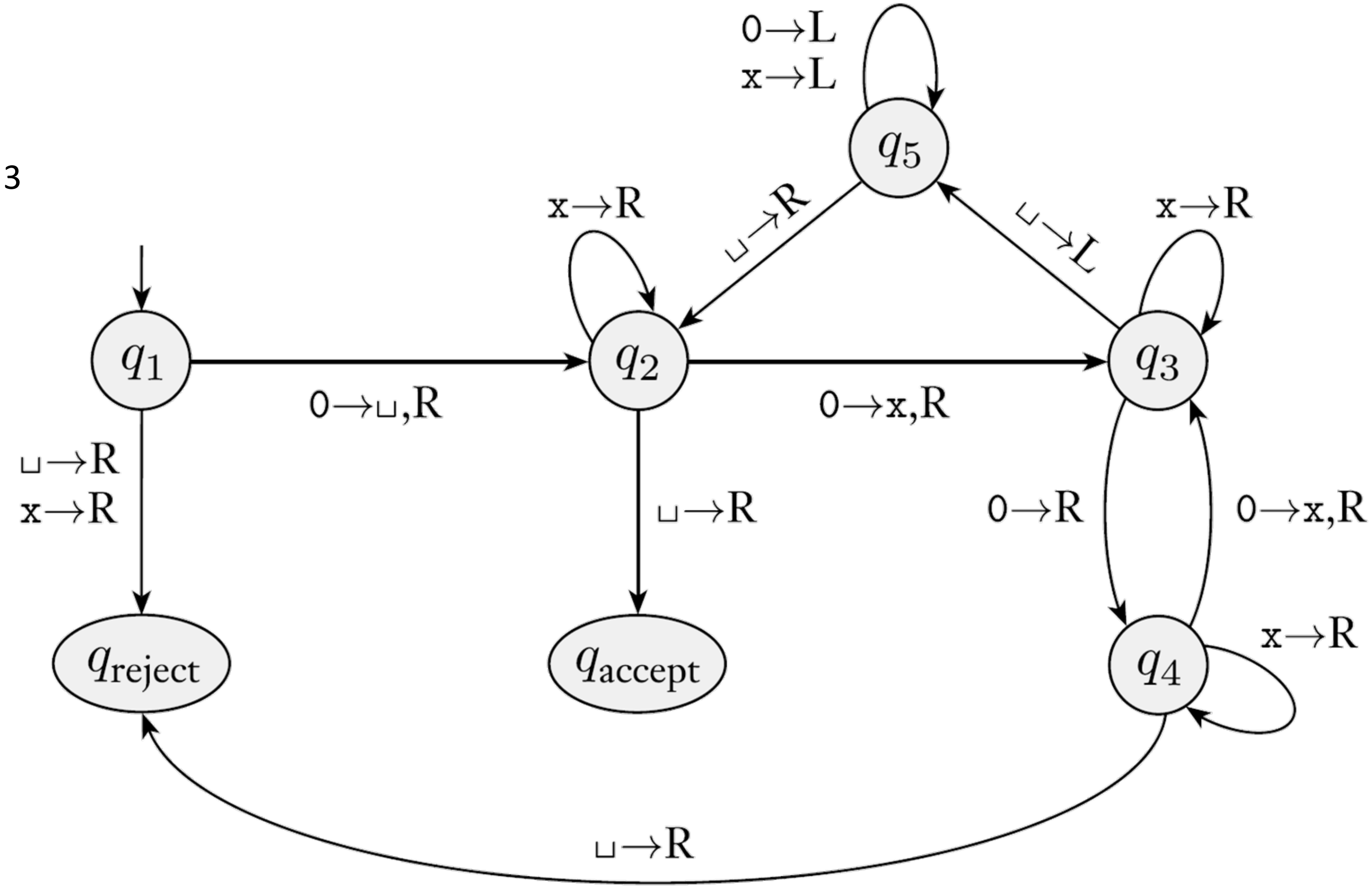


- Notation: a→b,D
  - a = symbol read on tape
  - b = symbol to be written on tape
    - May be left blank (does not alter tape)
  - D = direction the head moves
  - Shorthand for δ(q,a) = (r,b,D)

Example 1

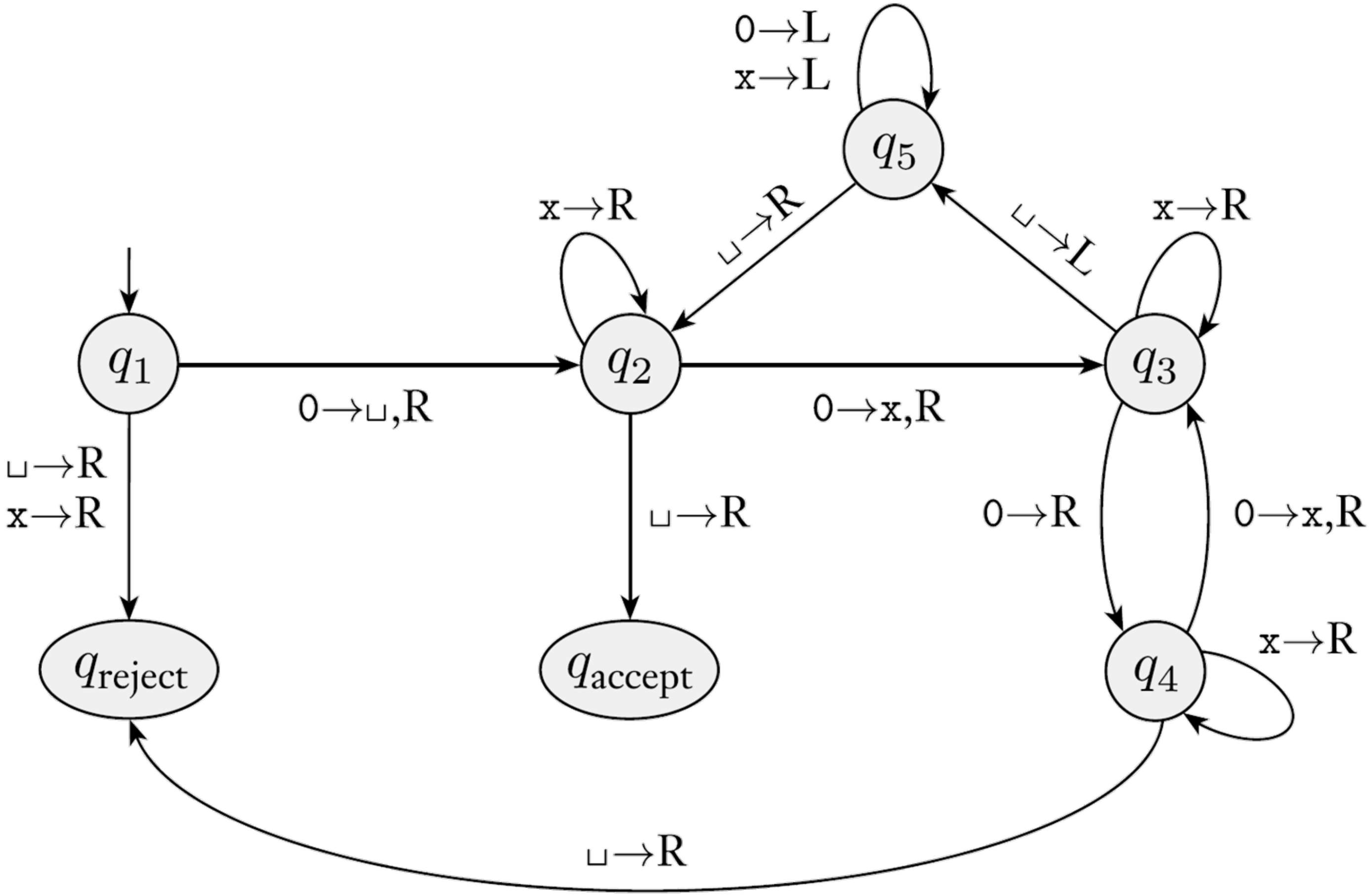
- Transitions
  - Begins by writing a blank on the leftmost 0
    - Marks the start of tape

  - State 2, checks if # of 0s is correct
    - If input = 0, marks x, moves right, go to q_3
    - If input = x, moves right
    - If input = ␣, moves right, go to q_{accept}
  - State 3, intermediate transition
    - If input = 0, moves right, go to q_4
    - If input = x, moves right
    - If input = ␣, moves left, go to q_5



Example 1

- Transitions
  - State 4, checks if # of 0s is incorrect
    - If input = 0, moves right, marks x, go to q_3
    - If input = x, moves right
    - If input = ␣, moves right, go to q_3
  - State 5, moves head back to left side
    - If input = 0, moves left
    - If input = x, moves left
    - If input = ␣, moves right, go to q_2



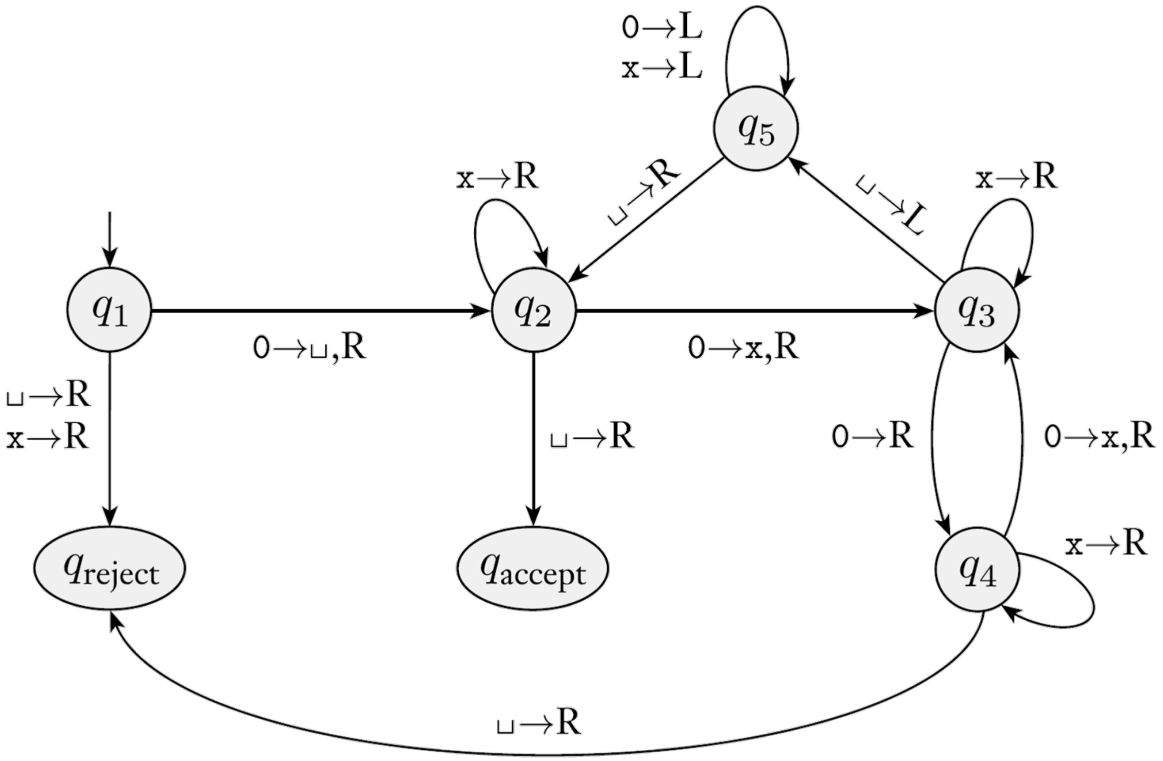
Example 1

- Sample input: 0000
  - Start configuration: $q_1 0000$

$q_1 0000$
$\sqcup q_2 000$
$\sqcup \mathtt{x} q_3 00$
$\sqcup \mathtt{x} 0 q_4 0$
$\sqcup \mathtt{x} 0 \mathtt{x} q_3 \sqcup$
$\sqcup \mathtt{x} 0 q_5 \mathtt{x} \sqcup$
$\sqcup \mathtt{x} q_5 0 \mathtt{x} \sqcup$

$\sqcup q_5 \mathtt{x} 0 \mathtt{x} \sqcup$
$q_5 \sqcup \mathtt{x} 0 \mathtt{x} \sqcup$
$\sqcup q_2 \mathtt{x} 0 \mathtt{x} \sqcup$
$\sqcup \mathtt{x} q_2 0 \mathtt{x} \sqcup$
$\sqcup \mathtt{xx} q_3 \mathtt{x} \sqcup$
$\sqcup \mathtt{xxx} q_3 \sqcup$
$\sqcup \mathtt{xx} q_5 \mathtt{x} \sqcup$

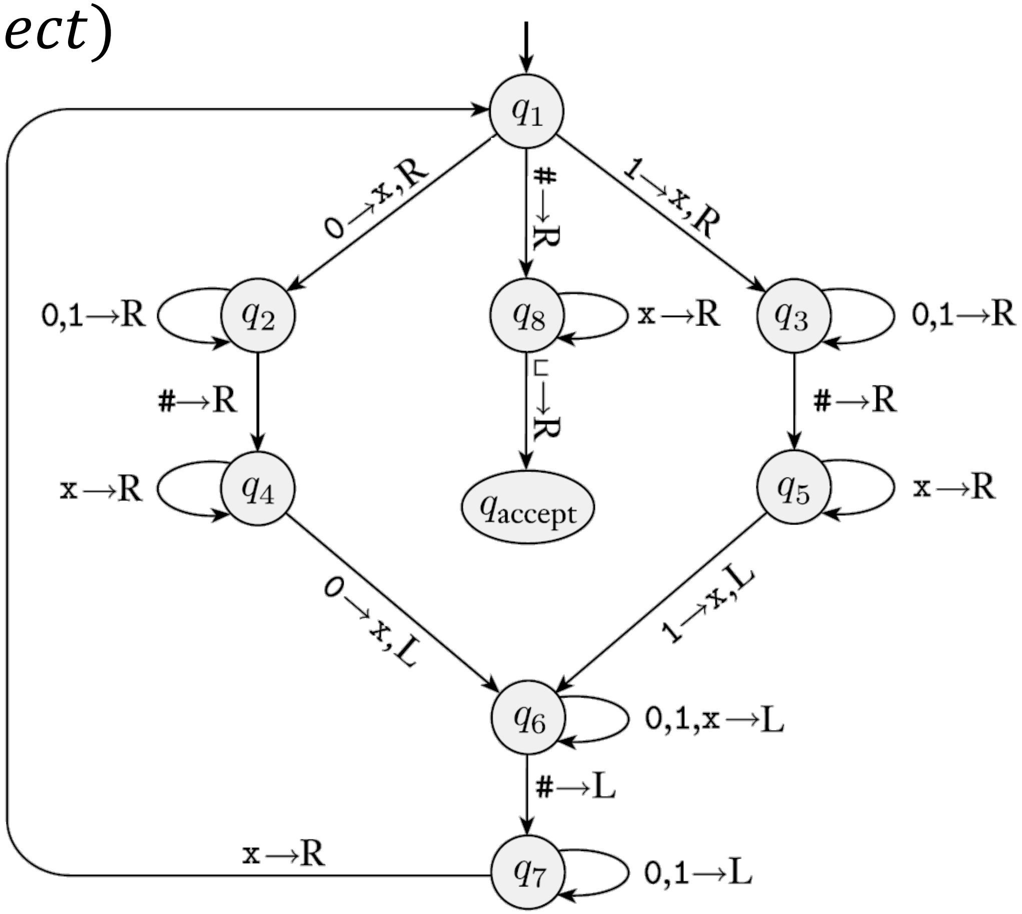
$\sqcup \mathtt{x} q_5 \mathtt{xx} \sqcup$
$\sqcup q_5 \mathtt{xxx} \sqcup$
$q_5 \sqcup \mathtt{xxx} \sqcup$
$\sqcup q_2 \mathtt{xxx} \sqcup$
$\sqcup \mathtt{x} q_2 \mathtt{xx} \sqcup$
$\sqcup \mathtt{xx} q_2 \mathtt{x} \sqcup$
$\sqcup \mathtt{xxx} q_2 \sqcup$
$\sqcup \mathtt{xxx} \sqcup q_{\mathrm{accept}}$



Example 2

- TM of previous lecture which decides the language B = {w#w | w ∈ {0,1}*}

- Formal Definition, $M = (Q, \Sigma, \Gamma, \delta, q_0, qaccept, qreject)$
  - $Q = \{q_1, \dots, q_8, q_{accept}, q_{reject}\}$
  - Σ={0,1,#}
  - $\Gamma = \{0, 1, \#, \mathrm{x}, \sqcup\}$
  - We describe δ with a state diagram
  - The start, accept, and reject states are q_1, q_{accept}, and q_{reject}, respectively



- Notation: a,b→D
  - Reads either a or b
  - Moves in the D direction

- Reject state is not shown
  - Rejects when reading symbol with out a transition

Example 3

- M_3 decides the language $C = \{a^i b^j c^k \mid i \; x \; j = k \; and \; i, j, k \geq 1\}$

- M_3 = "On input string w:

1. Scan input from left to right to check if single is a member of $a^+ b^+ c^+$
    - Reject if not a member

2. Return head to leftmost end

3. Cross off an a and scan right until a b occurs
    - Go back and forth between b's and c's while crossing off one of each until all b's are gone
    - If all c's are crossed off and some b's remain, reject

4. Restore crossed off b's and repeat stage 3 if there is another a to cross off
    - If all a's are crossed off and all c's are crossed off, accept
    - Otherwise, reject"

Example 4

- M_4 decides the language

$$E = \{\#x_1\#x_2\# \ldots \#xl \mid each\ xi \ \in \{0,1\} * \ and\ xi \neq x_j\ for\ each\ i \neq j\}$$

- M_4 = “On input w:
    1. Place a mark on top of the leftmost tape symbol (ex $\dot{\#}$).
        - If that symbol was a black, accept
        - If symbol was a #, continue with the next stage
        - Reject, otherwise
    2. Scan right to the next # and place a second mark on top of it
        - If no # is encountered before a blank symbol, only x_1 was present; accept
    3. Go back and forth between the two words on the right of the marked hashes
        - If they match reject
    4. Move the 2nd mark to the next hash symbol
        - If there is no more hash symbols on the right, then move the 1st hash symbol to its next one
            - Move the 2nd mark to the hash immediately after the 1st
            - If no hashes are available to move to, all words have been compared; accept
    5. Return to stage 3”