Mathematical Review

Sets

Sets

- A ***set*** is an unordered collection of objects.
  - the students in this class
  - the chairs in this room

- The objects in a set are called the ***elements***, or ***members*** of the set. A set is said to ***contain*** its elements.

- The notation $a \in A$
  - a is an element of the set A.

- The notation $a \notin A$
  - a is not a member of A

Describing a Set: Roster Method

- **Roster Method**
  - All members of a set are listed between braces.
  - $S = \{a,b,c,d\}$

- Order not important
  - $S = \{a,b,c,d\} = \{b,c,a,d\}$

- Each distinct object is either a member or not;
  - listing more than once does not change the set.
  - $S = \{a,b,c,d\} = \{a,b,c,b,c,d\}$

- Elipses (…) may be used to describe a set without listing all of the members when the pattern is clear.

  $S = \{a,b,c,d, \ldots,z\}$

Roster Method

- Set of all vowels in the English alphabet:

  $V = \{a,e,i,o,u\}$

- Set of all odd positive integers less than 10:

  $O = \{1,3,5,7,9\}$

- Set of all positive integers less than 100:

  $S = \{1,2,3,\ldots,99\}$

- Set of all integers less than 0:

  $S = \{\ldots, -3,-2,-1\}$

Some Important Sets

N = *natural numbers* = {0,1,2,3….}

Z = *integers* = {…,-3,-2,-1,0,1,2,3,…}

$\mathbf{Z}^+$ = *positive integers* = {1,2,3,…..}

R = set of *real numbers*

$\mathbf{R}^+$ = set of *positive real numbers*

C = set of *complex numbers.*

Q = set of rational numbers

Describing a Set: Set-Builder Notation

- Set-Builder Notation
  - Specify the property or properties that all members must satisfy:

    $S = \{x \mid x \text{ is a positive integer less than } 100\}$

    $O = \{x \mid x \text{ is an odd positive integer less than } 10\}$

    $O = \{x \in \mathbf{Z}^{+} \mid x \text{ is odd and } x < 10\}$

- A predicate may be used:

  $$S = \{x \mid P(x)\}$$

  - Example: $S = \{x \mid \text{Prime}(x)\}$

- Positive rational numbers:

  $\mathbf{Q}^{+} = \{x \in \mathbf{R} \mid x = p/q, \text{ for some positive integers } p,q\}$

Interval Notation

$$[a,b] = \{x \mid a \leq x \leq b\}$$
$$[a,b) = \{x \mid a \leq x < b\}$$
$$(a,b] = \{x \mid a < x \leq b\}$$
$$(a,b) = \{x \mid a < x < b\}$$

closed interval [a,b]

open interval (a,b)

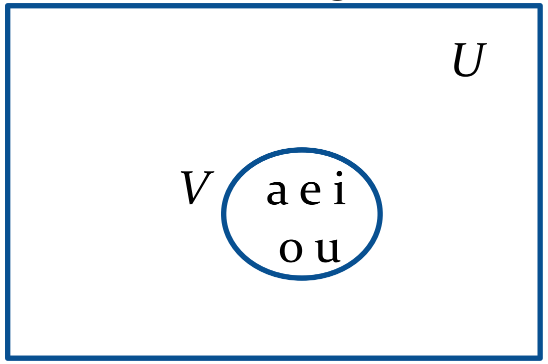
Universal Set and Empty Set

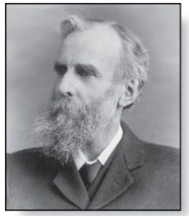
- Venn Diagram
  - Diagram of various shapes used to represent sets and elements.
  - Used to indicate the relationship between sets.

- The **universal set** U is the set containing everything currently under consideration.
  - Sometimes implicit
  - Sometimes explicitly stated.
  - Contents depend on the **context**.

- The **empty set** is the set with no elements.
  - Symbolized ∅, but {} also used.

Venn Diagram





John Venn (1834-1923)
Cambridge, UK

Set Equality

Definition: Two sets are *equal* if and only if they have the same elements.

- Therefore if A and B are sets, then A and B are equal if and only if

$$\forall x (x \in A \leftrightarrow x \in B)$$

- We write $A = B$ if A and B are equal sets.

$$\{1,3,5\} = \{3, 5, 1\}$$
$$\{1,5,5,5,3,3,1\} = \{1,3,5\}$$

Subsets

Definition: The set A is a *subset* of B, if and only if every element of A is also an element of B.

- The notation $A \subseteq B$ is used to indicate that A is a subset of the set B.

- $A \subseteq B$ holds if and only if $\forall x(x \in A \rightarrow x \in B)$ is true.
  1. Because $a \in \emptyset$ is always false, $\emptyset \subseteq S$,for every set S.
  2. Because $a \in S \rightarrow a \in S$, $S \subseteq S$, for every set S.

Tuples

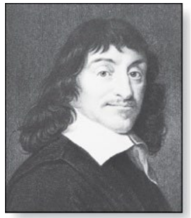
- The **ordered n-tuple** ($a_1, a_2, \ldots, a_n$) is the ordered collection that has a_1 as its first element and a_2 as its second element and so on until a_n as its last element.

- Two n-tuples are equal if and only if their corresponding elements are equal.

- 2-tuples are called **ordered pairs**.

- The ordered pairs (a,b) and (c,d) are equal if and only if $a = c$ and $b = d$.

Cartesian Product



René Descartes
(1596-1650)

Definition: The **Cartesian Product** of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Example:

$A = \{a,b\} \quad B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

- **Definition**: A subset R of the Cartesian product $A \times B$ is called a **relation** from the set A to the set B.

Cartesian Product

Definition: The cartesian products of the sets $A_1, A_2, \ldots\ldots, A_n$, denoted by $A_1 \times A_2 \times \ldots\ldots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots\ldots, a_n)$ where a_i belongs to A_i for i = 1, … n.

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i \text{ for } i = 1, 2, \ldots n\}$$

Example: What is $A \times B \times C$ where $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$

Solution: $A \times B \times C$ = {(0,1,0), (0,1,1), (0,1,2),(0,2,0), (0,2,1), (0,2,2),(1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)}